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The GUT Scale and Superpartner Masses from Anomaly Mediated Supersymmetry Breaking

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Abstract

We consider models of anomaly-mediated supersymmetry breaking (AMSB) in which the grand unification (GUT) scale is determined by the vacuum expectation value of a chiral superfield. If the anomaly-mediated contributions to the potential are balanced by gravitational-strength interactions, we find a model-independent prediction for the GUT scale of order $M_{\rm Planck}/(16\pi^2)$. The GUT threshold also affects superpartner masses, and can easily give rise to realistic predictions if the GUT gauge group is asymptotically free. We give an explicit example of a model with these features, in which the doublet-triplet splitting problem is solved. The resulting superpartner spectrum is very different from that of previously considered AMSB models, with gaugino masses typically unifying at the GUT scale.

1 Introduction

In supersymmetric extensions of the Standard Model, supersymmetry (SUSY) is broken in some separate sector of the theory and communicated to visible sector fields through gauge or gravitational interactions. Gravitational effects typically lead to superpartner masses from contact terms suppressed by powers of the Planck scale, giving rise to superpartner masses of order $m_{3/2}$. If these contact terms are absent, superpartner masses of order $m_{3/2}/(16\pi^2)$ are still generated due to the superconformal anomaly [1, 2]. This is called anomaly-mediated SUSY breaking (AMSB), and it naturally dominates if SUSY is broken on a separate brane [1, 3]. In AMSB, all soft masses are related to beta functions and anomalous dimensions, and are therefore completely determined by the quantum numbers of the relevant field up to the overall scale. In particular, squark masses other than the stop are determined to high accuracy by the gauge couplings, and therefore there are no dangerous flavor-changing neutral currents (FCNC's).

The minimal version of AMSB is obtained by simply coupling the minimal supersymmetric standard model (MSSM) to AMSB. However, this gives rise to negative slepton mass-squared terms at the weak scale due to the signs of the $SU(2)_W$ and $U(1)_Y$ beta functions. Modifying the superpartner spectrum is nontrivial because the AMSB predictions are largely insensitive to heavy supersymmetric thresholds. Several solutions to this problem have been suggested [4, 5, 6, 7, 8]. One is to change the effective theory at the weak scale to an extension of the MSSM [4]. Another possibility relies on the fact that any heavy threshold affects the soft terms beyond leading order in the SUSY breaking. If the SUSY-breaking splittings of a heavy multiplet are large, there can be large threshold corrections to visible sector sparticle masses [5]. Finally, Ref. [6] pointed out that if a heavy threshold is determined by a modulus field whose potential arises from SUSY breaking, the predictions for visible sparticle masses are changed. This latter mechanism is particularly attractive, and is the one we will employ in the present paper.

An obvious candidate for the heavy threshold is the grand unification (GUT) scale. In fact, we will show that there is a simple and robust mechanism in which AMSB gives rise to a VEV for a modulus field of order $M_{\text{Planck}}/(16\pi^2)$, precisely the right size of the GUT scale. (The mechanism is very similar to a mechanism first discussed in the context of gauge mediated models [9]. Other mechanisms for dynamically generating the GUT scale were considered in [10].) In addition, this threshold can lead to a realistic superpartner spectrum. We present an explicit model that incorporates these features, while also explaining doublet-triplet splitting.

The phenomenology of this class of models is very interesting. The soft SUSY breaking masses can be obtained by starting with the AMSB soft masses at the GUT scale, and then running them down to the weak scale without large threshold corrections at the GUT scale.[1] Therefore, the gaugino masses generally unify at the GUT scale as in traditional hidden sector models or gauge-mediated models.

One constraint on this class of models is that extra structure is required to avoid FCNC's. The problem is that dimension-4 Kähler terms can mix the modulus field with visible sector fields, giving rise to visible sector scalar masses of order $\langle X \rangle^2 m_{3/2}^2 / M_{\text{Planck}}^2$, where $\langle X \rangle$ is the modulus VEV. For $\langle X \rangle \sim M_{\text{GUT}} \sim 10^{16}$ GeV, this is as large as the AMSB contribution, and there is no reason for this contribution to conserve flavor. This constraint applies to any model with a large modulus VEV that is determined by SUSY breaking. We will show that these terms can be naturally suppressed by assuming that the visible sector gauge and Higgs fields live on a 'thick brane', with the hidden sector and the visible sector matter fields localized at different positions within the brane.[2] Alternatively, one can simply assume that the unwanted dimension-4 terms are suppressed; this does not appear as unnatural as the fine-tuning of conventional hidden-sector models, since we assume that *all* Kähler terms of a certain structure are small.

Because of the constraint above, we also briefly consider mechanisms that give lower thresholds. These can be motivated by GUT's with intermediate scales. The results depend on the details of the models, but we conclude that there is a large class of interesting models.

This paper is organized as follows. In Section 2, we review some basic facts about AMSB and the non-decoupling of heavy thersholds that are determined by moduli. In Section 3, we explain our mechanism of obtaining the GUT scale dynamically from the Planck scale. We also discuss several alternatives for generating a non-decoupling threshold. We construct a specific model in Section 4. Several possible mechanisms for generating a μ term are described in Section 5. We discuss phenomenology in Section 6. Our conclusions are in Section 7.

[1] If the GUT threshold were supersymmetric, the threshold corrections would be large, resulting in minimal AMSB predictions for the low-energy theory.

[2] The fact that 'thick branes' can modify naturalness in an interesting way was pointed out in Ref. [11].

2 AMSB and Decoupling

If SUSY is broken in a sector that communicates with the visible sector only via gravity, and there are no contact interactions between the hidden and visible sector, then SUSY breaking will be communicated by anomaly mediation. In the visible sector, all SUSY breaking effects enter through the supergravity chiral compensator field

$$\phi = 1 + \theta^2 F_\phi, \tag{1}$$

where $F_\phi \sim m_{3/2}$. The couplings of ϕ are restricted by the fact that ϕ is chiral, and by a spurion scale symmetry under which ϕ has mass dimension +1. Therefore, ϕ only appears in terms with dimensionful couplings. For example, a chiral mass term is covariantized by the replacement $M \to M\phi$.

If the visible sector does not contain dimensionful couplings, there is no SUSY breaking at tree level. However, in a supersymmetric regulator, the cutoff is a dimensionful coupling that must be covariantized. This means that there are soft masses at loop level. Writing the Lagrangian for the visible sector

$$\mathcal{L}_{\text{vis}} = \int d^4\theta\, Z(\mu/\Lambda) Q^\dagger e^V Q + \left(\int d^2\theta \left[S(\mu/\Lambda) W^\alpha W_\alpha + \lambda Q^3 \right] + \text{h.c.} \right), \tag{2}$$

we replace

$$Z\left(\frac{\mu}{\Lambda}\right) \to Z\left(\frac{\mu}{\Lambda(\phi^\dagger \phi)^{1/2}}\right), \qquad S\left(\frac{\mu}{\Lambda}\right) \to S\left(\frac{\mu}{\Lambda\phi}\right). \tag{3}$$

Expanding in θ, one finds the AMSB soft masses[3]

$$\begin{aligned} m_0^2(\mu) &= -\frac{1}{4} \frac{\partial \gamma(\mu)}{\partial \ln \mu} |F_\phi|^2 \, , \\ m_{1/2}(\mu) &= \frac{\beta(\mu)}{g} F_\phi \, , \\ A(\mu) &= -\frac{1}{2} \gamma(\mu) F_\phi \, . \end{aligned} \tag{4}$$

Here β is the gauge beta function and $\gamma = \partial \ln Z / \partial \ln \mu$ is the anomalous dimension.

It is not hard to see why a supersymmetric threshold does not affect the soft scalar masses. For example, the gaugino masses at one loop are parameterized by the chiral superfield

$$S = \frac{1}{2g^2} + \frac{m_\lambda}{g^2} \theta^2 \, . \tag{5}$$

[3]This is analogous to the method of Giudice and Rattazzi [12] for extracting soft masses in models of gauge mediated supersymmetry breaking.

The effective gauge coupling below a SUSY threshold M is given by

$$S_{\rm eff}(\mu) = S_0 + \frac{b}{16\pi^2}\ln\frac{M}{\Lambda} + \frac{b_{\rm eff}}{16\pi^2}\ln\frac{\mu}{M}\ , \tag{6}$$

where b and $b_{\rm eff}$ are the beta function coefficients in the theory above and below the scale M, respectively. To covariantize, both M and Λ are rescaled by ϕ:

$$S_{\rm eff}(\mu) \to S_0 + \frac{b}{16\pi^2}\ln\frac{M}{\Lambda} + \frac{b_{\rm eff}}{16\pi^2}\ln\frac{\mu}{M\phi}\ . \tag{7}$$

We see that the soft masses depend only on the beta function in the effective theory below M. This result generalizes to all couplings to all orders in perturbation theory: the effective coupling is a function of the ratios $\mu/M \to \mu/(M\phi)$ and $M/\Lambda \to M/\Lambda$. The threshold M acts as the cutoff for the effective theory, and the soft masses are insensitive to the presence of the heavy threshold.[4]

It is worth reviewing how these results arise in terms of component diagrams. An explicit SUSY mass term M will give rise to a SUSY breaking B-type mass-squared term MF_ϕ. Loop threshold corrections at the scale M therefore give SUSY breaking masses of order $F_\phi/(16\pi^2)$, precisely the size of the AMSB contributions. These corrections ensure that the low-energy SUSY breaking masses coincide with the AMSB predictions appropriate to the effective theory below the scale M.

The results are completely different if the threshold is due to the VEV of a light field whose potential arises from SUSY breaking [6]. The models we consider have a field X with superpotential couplings of the form

$$\Delta W = \lambda X T_1 T_2. \tag{8}$$

In the SUSY limit, there is a flat direction with $X \neq 0$ along which the fields $T_{1,2}$ get masses λX. Integrating out $T_{1,2}$ at the scale $\langle X\rangle$ we can read the gaugino mass off the low-energy gauge coupling function

$$S_{\rm eff}(\mu) \to S_0 + \frac{b}{16\pi^2}\ln\frac{\lambda X}{\Lambda\phi} + \frac{b_{\rm eff}}{16\pi^2}\ln\frac{\mu}{\lambda X}\ . \tag{9}$$

Since X is a field, it is not rescaled by ϕ, so that $\langle F_X\rangle \neq \langle X\rangle\langle F_\phi\rangle$, and the AMSB prediction is modified. Models of this type are considered in Ref. [6].

As we explain in the next Section, in the models we construct $\langle F_X\rangle/\langle X\rangle \ll F_\phi$. Then, the soft masses at the weak scale can be obtained by calculating the AMSB

[4] This only holds to leading order in the SUSY breaking. The soft terms typically receive corrections of order F^4/M^2 and higher due to the heavy threshold. These corrections are of course only relevant for small M.[5]

soft masses in the full theory at the scale $\langle X \rangle$ and running down to the weak scale. This can be seen simply by considering the component calculation of the soft masses. The fields that are integrated out at the GUT scale have SUSY masses of order $\langle X \rangle$, and SUSY breaking mass-squared terms of order $\langle F_X \rangle$. Because $\langle F_X \rangle / \langle X \rangle \ll F_\phi$, integrating out these fields does not give large loop matching corrections, and tree level matching with 1-loop running gives a good approximation to the low-energy masses.

3 The GUT Modulus

We now describe the mechanism for generating the GUT scale. We consider GUT models with a flat direction along which the GUT gauge group is spontaneously broken to the Standard-Model gauge group. We also require that AMSB give a negative mass-squared to this flat direction; we will discuss this point further below. If there are no other contributions to the potential for the flat direction, the theory will run away, presumably giving a noninteracting theory. However, the potential can be stabilized by Kähler terms of the form

$$\Delta \mathcal{L} = \int d^4\theta \, \frac{c}{M_{\text{Planck}}^2 \phi^\dagger \phi} (\Sigma^\dagger \Sigma)^2 \ , \tag{10}$$

where Σ is a field with a nonzero VEV along the flat direction. This gives a contribution to the potential for Σ

$$\Delta V(\Sigma) = -\frac{c|F_\phi|^2}{M_{\text{Planck}}^2} (\Sigma^\dagger \Sigma)^2 \ . \tag{11}$$

For $c < 0$ this can stabilize the VEV at

$$\langle \Sigma \rangle \sim \left(\frac{M_{\text{Planck}}^2 m_\Sigma^2}{c|F_\phi|^2} \right)^{1/2} \sim \frac{M_{\text{Planck}}}{16\pi^2} \ , \tag{12}$$

where $m_\Sigma^2 \sim |F_\phi|^2/(16\pi^2)^2$ is the AMSB soft mass of Σ. Note that this automatically generates a VEV at the GUT scale, assuming only that all couplings are order 1. The correct value of the GUT scale is generated from the Planck scale purely by supergravity effects!

AMSB dominates only if there are no large contact terms between the hidden and observable sector. This is natural if the hidden and observable sector are localized on different branes, and separated by a distance that is larger than the fundamental Planck scale. But if there are large extra dimensions, the size of higher-dimension

operators such as Eq. (10) is set by the fundamental Planck scale, which is smaller than the 4-dimensional Planck scale. For one extra dimension, this relation is

$$M_{\text{Planck}}^2 = 2\pi r M_5^3 \ , \tag{13}$$

where r is the radius of the extra dimension. The unwanted direct contact terms have the form

$$\Delta\mathcal{L} = \int d^4\theta \, \frac{a_{jk}}{M_5^2} N^\dagger N Q_j^\dagger Q_k \ , \tag{14}$$

where N is the hidden sector field that breaks SUSY via $\langle F_N \rangle \neq 0$, and $a \sim e^{-M_5 r}$. It is easy to see that for $r \sim 10/M_5$ we can suppress the unwanted operators and still obtain a reasonable value of the GUT scale via the mechanism above.

We now turn to the question of how the threshold at $\langle \Sigma \rangle$ affects the SUSY breaking masses. The relevant quantity is $\langle F_\Sigma \rangle / \langle \Sigma \rangle$, since this determines the size of the SUSY breaking corrections from loops involving GUT scale fields. The effective theory below the scale $\langle \Sigma \rangle$ contains a light Standard Model singlet field that parametrizes the motion along the flat direction. We label this field by X. (In the fundamental theory, the flat direction typically corresponds to a combination of fields, some of which have GUT charges.) In the effective theory, the potential for X is given by

$$\Delta\mathcal{L}_{\text{eff}} = \int d^4\theta \left[Z_X\Big(\mu = (X^\dagger X)^{1/2}\Big)\, X^\dagger X + \frac{c}{M_{\text{Planck}}^2 \phi^\dagger \phi} (X^\dagger X)^2 \right] . \tag{15}$$

The wavefunction factor for X is evaluated at the scale $\mu = (X^\dagger X)^{1/2}$ because X has only irrelevant interactions below the scale $\langle \Sigma \rangle$ where the GUT fields are integrated out. $\langle F_X \rangle$ is determined by expanding the first term

$$\Delta\mathcal{L}_{\text{eff}} \sim F_X^\dagger F_X + \left(F_X^\dagger \frac{X\, F_\phi}{16\pi^2} + \text{h.c.} \right) \ , \tag{16}$$

which gives

$$\frac{\langle F_X \rangle}{\langle X \rangle} \sim \frac{F_\phi}{16\pi^2} \ . \tag{17}$$

Since $\langle F_X \rangle \ll \langle X \rangle F_\phi$, we can neglect the contributions from X as a messenger field compared to the contributions from AMSB. As discussed earlier, the soft masses can therefore be computed by running the AMSB masses at the GUT scale down to the weak scale.[5]

[5]The renormalization group running is also suppressed by a factor of $1/(16\pi^2)$, but is enhanced by large logs. There are no large logs in the X messenger contribution, since the parameters in Eq. (15) are not renormalized below the GUT scale.

The gauge contribution to the one-loop running of the scalar masses squared is always positive. However, the positive running contribution to the right-handed slepton masses is very small, so we are led to require that the scalar masses are positive at the GUT scale. The AMSB contribution to scalar masses has the schematic form

$$m^2 \sim \left(\frac{1}{16\pi^2}\right)^2 \left[+y^2(y^2-g^2) \pm g^4\right] F_\phi^2, \tag{18}$$

where y is a Yukawa coupling and g is a gauge coupling. The last term is positive if the gauge group is asymptotically free, and negative if it is infrared free. For the first two generations, the Yukawa couplings are small.[6] We therefore require the slepton fields to be charged under an asymptotically free gauge group. The only possibilities are a gauged non-abelian horizontal symmetry, or the GUT group itself. However, the number of GUT fields is so large that a gauged horizontal symmetry will not be asymptotically free. We are therefore led to the requirement that the GUT group be asymptotically free. It is nontrivial to satisfy this requirement in a realistic GUT, but we will give an example in the next section that shows that it is possible.

If the GUT group is asymptotically free, then the g^4 contribution to the mass-squared of the GUT modulus is also positive, while our mechanism requires a negative value. We can obtain a negative mass-squared for the flat direction either from the $-y^2g^2$ term in Eq. (18), or by charging some of the fields along the flat direction under an infrared-free gauge group (such as a $U(1)$ factor). Both of these effects are present in the model of Section 4, and give a negative mass-squared for the flat direction.

Since it is difficult to construct a realistic asymptotically free $SU(5)$ GUT model, larger groups are typically needed. Thus, the Standard Model fields will generically be charged under some broken diagonal generators of the GUT group. As discussed in [6] this can give rise to additional D-term contributions to the soft masses. These contributions can have important phenomenological consequences, but they are highly model dependent. We note that it may be possible to construct a model in which D-term contributions to slepton masses squared are sufficiently positive, so that the GUT group does not have to be asymptotically free. For this to work, all sleptons must have same-sign charges under the appropriate $U(1)$. We do not explore this possibilty here.

An important constraint on this scenario is that we must forbid couplings of the form

$$\Delta\mathcal{L} = \int d^4\theta \frac{c_{jk}}{M_{\text{Planck}}^2 \phi^\dagger\phi} (\Sigma^\dagger\Sigma)(Q_j^\dagger Q_k)\ , \tag{19}$$

[6]One could attempt to build models in which the leptons have large Yukawa couplings with exotic fields above the GUT scale. We will not pursue this possibility here.

where the Q's are MSSM matter fields. These terms cannot be forbidden by any symmetries, and there is no reason that they should conserve flavor. They give rise to contributions to scalar masses of order

$$\Delta m_{jk}^2 \sim \frac{c_{jk}|F_\phi|^2}{M_{\rm Planck}^2} \langle\Sigma\rangle^\dagger \langle\Sigma\rangle \sim c_{jk} \left(\frac{1}{16\pi^2}\right)^2 |F_\phi|^2 \ , \tag{20}$$

where we have used $\langle\Sigma\rangle \sim M_{\rm Planck}/(16\pi^2)$. These contributions are precisely the size of the AMSB contributions, ruining the natural absence of FCNC's. This fine-tuning is in a sense less severe than in traditional hidden-sector models, because we require *all* operators of the form Eq. (19) to be small, and not just a special subset of them (the flavor non-diagonal ones).

We can suppress the dangerous contributions without fine tuning by localizing the GUT modulus fields Σ and the visible sector fields Q on different branes. Locality then guarantees the absence of terms such as Eq. (19). Gauge fields must couple to both Σ and Q, and must therefore live in the bulk. Doublet-triplet splitting typically requires that the Standard Model Higgs fields couple to Σ (the field that breaks the GUT group) as well as to Q. The Standard Model Higgs fields must therefore also live in the bulk. The simplest version of these ideas therefore allows the Standard Model gauge and Higgs fields to couple directly to the hidden sector where SUSY is broken. This gives soft terms that are large compared to the AMSB contributions.[7]

The way out is to assume that the Standard Model gauge and Higgs fields are localized on a 'thick brane', with the matter fields Q and GUT Higgs fields Σ localized at different positions within the thick brane. The hidden sector is assumed to be localized on a brane outside the thick brane. Now (approximate) locality suppresses all contact terms between the hidden and visible sector, while also suppressing the undesirable couplings among the visible sector fields. This structure arises in simple models with scalar domain walls [11].[8]

We close this section by outlining a few alternatives for dynamically generating a non-decoupling threshold. The first is to generate $m_\Sigma^2 < 0$ as above, but to stabilize Σ with superpotential terms of the form

$$\Delta W \sim \frac{\Sigma^n}{M_{\rm Planck}^{n-3}} \ .$$

(Here we loosely treat Σ as a gauge-singlet for simplicity. In concrete models Σ is typically replaced by a pair of fields which parameterize a D-flat direction.) Such a

[7]Putting the gauginos and Higgs fields in the bulk and allowing direct couplings to the hidden sector results in 'gaugino mediated SUSY breaking' [14], which gives a natural and interesting scenario. However, we are interested in exploring the possibility that AMSB dominates.

[8]For further discussion of SUSY models involving thick branes see for example Ref. [13].

superpotential term leads to a scalar potential contribution of the form

$$\Delta V \sim \frac{\Sigma^{2(n-1)}}{M_{\text{Planck}}^{2(n-3)}} + F_\Sigma \frac{\Sigma^n}{M_{\text{Planck}}^{n-3}} \ . \tag{21}$$

+ The second term in (21) is subdominant, and the first term would stabilize the VEV at

$$\langle \Sigma \rangle \sim 10^{-15/(n-2)}\, M_{\text{Planck}} \ .$$

For example, for $n = 4$ we obtain $\langle \Sigma \rangle \sim 10^{11}$ GeV. Such superpotential terms may therefore be used to generate a threshold in models of intermediate scale unification. We also note that in order to rely on our basic mechanism of using Kähler potential terms to generate the GUT scale, superpotential terms of the form (21) should be forbidden by some symmetry for $n \sim 9$ or smaller, since otherwise they would lead to a VEV smaller than $M_{\text{GUT}} \sim 10^{16}$ GeV.

Finally, another way to generate a non-decoupling threshold is to use a strongly-coupled theory in which a runaway potential is generated dynamically, driving some field to infinity. This field may be stabilized by a positive AMSB scalar mass-squared. In this case, the resulting VEV is related to the SUSY breaking scale as well as the strong coupling scale of the theory.

4 The '5–3–1' Model

We now construct a simple model realizing the ideas discussed in the previous section. The most important model building constraint arises from the requirement that the Standard Model fields are charged under an asymptotically free gauge group above the GUT scale. This requirement would be satisfied, for example, in the simple $SU(5)$ model with only a GUT modulus in addition to the MSSM fields. However, in any GUT model with an $SU(5)$ factor one needs to generate a GUT-scale mass for the Higgs triplets. This requires introducing a number of additional fields, and generically leads to loss of asymptotic freedom. A simple solution to the doublet-triplet splitting problem that maintains asymptotic freedom can be obtained by giving up unification in the strictest sense. We follow Ref. [15] and consider a model with an $SU(5) \times SU(3) \times U(1)$ gauge group that is spontaneously broken to a 'diagonal' standard-model subgroup. The predictions of unification are recovered if the gauge couplings of the $SU(3)$ and $U(1)$ factors are sufficiently larger than the $SU(5)$ gauge coupling.

The MSSM fields are charged only under the $SU(5)$ factor. In addition, the model contains the fields of Table 1. Note that the $SU(5)$ factor is asymptotically

	$SU(5)$	$SU(3)$	$U(1)$
Σ	5	$\bar{3}$	1
$\bar{\Sigma}$	$\bar{5}$	3	-1
Δ	1	8	0
T	1	3	-1
$\bar{T}$	1	$\bar{3}$	1

Table 1: Field content of the '5–3–1' model. The standard-model matter fields are charged under $SU(5)$.

free, so AMSB masses for sleptons and squarks are all positive at the GUT scale. The superpotential is

$$W = \lambda_1 \bar{\Sigma}\Delta\Sigma + \lambda_2 \bar{\Sigma} H \bar{T} + \lambda_3 \Sigma \bar{H} T \, . \tag{22}$$

This model has a one-parameter flat direction with

$$\langle\Sigma\rangle, \langle\bar{\Sigma}\rangle \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \tag{23}$$

and all other fields vanishing. This breaks $SU(5) \times SU(3) \times U(1)$ down to the Standard Model gauge group, and this flat direction therefore acts as the GUT modulus in this model. The role of the other fields is as follows. H and $\bar{H}$ transform as a 5 and a $\bar{5}$ of $SU(5)$ and contain the Standard Model Higgs doublets. The fields T and $\bar{T}$ are $SU(3)$ triplets that get masses with the triplet components of H and $\bar{H}$ along the flat direction (the 'missing partner' mechanism). The field Δ is required to give masses to unwanted light fields. In the absence of the Δ term, there are unwanted Nambu-Goldstone modes due to accidental global symmetries of the superpotential. With the couplings above, the only light fields are the Standard Model fields and a single flat direction that can be parameterized by $\mathrm{tr}(\Sigma\bar{\Sigma})$. (For $\Sigma = 0$ there are also flat directions with $\Delta \neq 0$, but these do not affect the properties of vacua with $\Sigma \neq 0$.)

Unification to 5% accuracy requires the $U(1)$ gauge coupling to satisfy $g_1'(M_{\mathrm{GUT}}) \geq 0.8$. (We normalize g_1' so that $D_\mu = \partial_\mu - i g_1' X$, where X is the $U(1)$ charge defined in Table 1.) For this value of g_1', the $U(1)$ Landau pole is at $30 M_{\mathrm{GUT}}$. This is uncomfortable, but plausibly large enough to trust perturbation theory at the GUT scale.

We now consider the effect of SUSY breaking on the GUT modulus flat direction. The g_5^4 contribution to the mass-squared is positive because $SU(5)$ is asymptotically free, but there are several negative contributions that can offset this. First, note that the g_1^4 contribution is negative, and can naturally be larger than the g_5^4 contribution because g_1 is required to be large for unification. (Even at the lower limit of g_1, the contributions are comparable.) The $SU(3)$ factor can also give a negative contribution due to a fortunate accident of this model: the one-loop beta function for $SU(3)$ vanishes, so there is no g_3^4 term, and the negative $y^2 g_3^2$ term can naturally dominate the positive y^4 term if g_3 is larger than y. We conclude that one can obtain a negative mass-squared for the flat direction for a wide range of parameters.

The potential for the flat direction can be stabilized by the Kähler terms

$$\Delta\mathcal{L} = \int d^4\theta \, \frac{c}{M_{\rm Planck}^2 \phi^\dagger \phi} (\Sigma^\dagger \Sigma)^2 + \frac{\bar{c}}{M_{\rm Planck}^2 \phi^\dagger \phi} (\bar{\Sigma}^\dagger \bar{\Sigma})^2 \ , \tag{24}$$

with $c, \bar{c} \sim 1$. As discussed in Section 3, for $c, \bar{c} < 0$ this gives a stable minimum at

$$\langle X \rangle \sim \frac{1}{16\pi^2} M_{\rm Planck} \ . \tag{25}$$

As discussed in the previous section, superpotential terms of the form $(\bar{\Sigma}\Sigma)^n / M_{\rm Planck}^{2n-3}$ must be forbidden for $n < 5$. We note that one can impose an R symmetry to forbid such operators. Such a symmetry can only be made anomaly free if the Standard Model fields transform non-trivially under the symmetry and some of the Standard Model Yukawa couplings are generated after spontaneous breakdown of the R symmetry. This is plausible since the Standard Model Yukawas are small. One could also forbid dangerous operators by imposing discrete symmetries.

So far we have neglected D-term contributions. However, there are various effects that can give $\langle \Sigma \rangle \neq \langle \bar{\Sigma} \rangle$. First, note that there is a nonzero AMSB contribution to $m_\Sigma^2 - m_{\bar{\Sigma}}^2$ due to the top-quark Yukawa coupling, and also if $\lambda_2 \neq \lambda_3$:

$$m_\Sigma^2 - m_{\bar{\Sigma}}^2 \sim g_1^2 (\lambda_2^2 - \lambda_3^2) + \lambda_2^2 y_t^2. \tag{26}$$

The large g_1^4 contributions cancel in the difference, so it is not unnatural to have $m_\Sigma^2 - m_{\bar{\Sigma}}^2 \ll m_\Sigma^2 + m_{\bar{\Sigma}}^2$. Another contribution to the difference between $\langle \Sigma \rangle$ and $\langle \bar{\Sigma} \rangle$ arises from an asymmetry in the Kähler potential couplings $c \neq \bar{c}$.

The only non-vanishing D term in our model is associated with the broken $U(1)$ corresponding to the linear combination of the original $U(1)$ of the model and the hypercharge subgroup of $SU(5)$.[9] Thus, MSSM scalars receive additional contributions to their soft masses-squared proportional to their hypercharge. One should

[9] D-terms for other broken diagonal generators vanish due to the tracelessness of the group gener-

therefore check that these contributions do not drive the scalar masses negative. The largest dangerous contribution is the one associated with either the left-handed or the right-handed sleptons. It is given by

$$m_D^2 = Y_\ell \frac{g_5^2}{g_1^2 + g_5^2} \left[m_\Sigma^2 - m_{\bar{\Sigma}}^2 + \frac{c - \bar{c}}{c + \bar{c}} (m_\Sigma^2 + m_{\bar{\Sigma}}^2) \right] , \tag{27}$$

where Y_ℓ is the slepton hypercharge. If we require unification with 5% accuracy, we find

$$\frac{m_D^2}{m_{\rm AMSB}^2} = \left(\frac{m_\Sigma^2 - m_{\bar{\Sigma}}^2}{m_\Sigma^2 + m_{\bar{\Sigma}}^2} + \frac{c - \bar{c}}{c + \bar{c}} \right) k \, , \tag{28}$$

where $m_{\rm AMSB}^2$ is the usual AMSB mass-squared and $k \simeq 1$ for both right-handed and left-handed sleptons. We therefore conclude that in a large region of parameter space, our model predicts positive slepton masses at the GUT scale.

5 The μ Problem

No supersymmetric model is complete without a solution to the 'μ problem.' In this Section, we briefly consider several approaches in the present class of models.

The first is the mechanism proposed in Ref. [6]. In this mechanism, one posits a singlet S that gets its mass from the modulus X. If this singlet has superpotential couplings of the form

$$\Delta W = \lambda S H_u H_d \tag{29}$$

this will generate a μ term and a $B\mu$ term of the correct size. As discussed in Ref. [6], there is no danger of generating a large B term because there is no F-type VEV larger than $F_\phi/(16\pi^2)$.

This mechanism can be adapted to the present class of models. In order for the GUT modulus to give a GUT-scale mass to a singlet, we add the superpotential terms

$$\Delta W \sim N(\bar{\Sigma}\Sigma - Y^2) + Y S^2 + S \bar{H} H + S^3 \, , \tag{30}$$

where N, Y, S are singlets. The first term forces $Y \neq 0$ along the flat direction, and the second term gives the singlet S a mass as desired.

Another possibility for generating the μ term is to add a singlet S with superpotential couplings at the weak scale

$$W = \lambda S H_u H_d + \frac{k}{3} S^3 \, . \tag{31}$$

ators and the fact that the modulus VEV is proportional to the unit matrix under the corresponding subgroup. For example, for the $SU(3)$ generator $T_8 = \mathrm{diag}(1, 1, -2)$ one has $\Sigma^\dagger T_8 \Sigma = 0$, independently of the value of $\bar{\Sigma}$.

If S gets a VEV of order the weak scale, this can generate a μ term. This model is appealing because it can relax the upper bound on the lightest Higgs mass.

We can obtain a realistic version of this model in the context of the '5–3–1' model presented above. We assume that the superpotential at the GUT scale (see Eq. (22)) includes the additional terms

$$\Delta W = \lambda S \bar{H} H + \frac{k}{3} S^3 + \lambda' S \bar{T} T \ . \tag{32}$$

It is easy to check that there is still a one-parameter flat direction with $\langle \Sigma \rangle, \langle \bar{\Sigma} \rangle$ nonzero, and $\langle S \rangle = 0$, in the approximation that SUSY is unbroken. When SUSY is broken, the field S will get a VEV at the weak scale if $m_S^2 < 0$ at the weak scale. (A terms will also tend to destabilize $S = 0$.) The $S\bar{T}T$ coupling gives rise to a contribution $\Delta m_S^2 \sim -\lambda'^2 g_3'^2$ at the GUT scale, and so m_S^2 can have any value at the GUT scale. Similar couplings can presumably be found in other GUT models. It appears that there is no obstacle in constructing a realistic model of this type.

Finally, one can generate a VEV for S in (31), and hence, a μ term, following the mechanism of ref. [5]. In this mechanism, S is coupled to heavy fields of mass F_ϕ/λ', where λ' is a small number. As was shown in [5], such a mass is easily generated from AMSB, leading to a μ term and a B term of the correct size.

6 Phenomenology

We now turn to the superpartner mass spectrum. Our analysis applies not just to the '5–3–1' model of Section 4, but to general AMSB GUT models with the GUT scale determined by AMSB.

If the GUT gauge group is simple, the gaugino masses at the weak scale can be derived by noting that m_λ / g^2 is RG invariant to one loop, and using the GUT matching condition:

$$m_{\lambda i}(\mu) = g_i^2(\mu) \frac{\beta_{\rm GUT}}{g_{\rm GUT}^3} F_\phi \ . \tag{33}$$

Here $i = 1, 2, 3$ runs over the Standard Model group factors (with $SU(5)$ normalization for g_1) and

$$\beta_{\rm GUT} = \left. \frac{d g_{\rm GUT}}{d \ln \mu} \right|_{\mu = M_{\rm GUT}} . \tag{34}$$

In the '5–3–1' model the prediction for the gaugino masses is modified by the extra group factors. The matching conditions at the GUT scale are

$$\frac{m_{\lambda 3}}{g_3^2} = \frac{m_{\lambda 5}}{g_5^2} + \frac{m_{\lambda 3'}}{g_3'^2} \ , \quad m_{\lambda 2} = m_{\lambda 5} \ , \quad \frac{m_{\lambda 1}}{g_1^2} = \frac{m_{\lambda 5}}{g_5^2} + \frac{m_{\lambda 1'}}{15 g_1'^2} \ , \tag{35}$$

where g_3' and g_1' are the $SU(3)$ and $U(1)$ couplings in the '5–3–1' model. We therefore recover the simple GUT prediction Eq. (33) for the $SU(2)$ gaugino mass, while

$$m_{\lambda 3}(\mu) \;=\; g_3^2(\mu)\left[\frac{\beta_5(M_{\rm GUT})}{g_5^3(M_{\rm GUT})} + \frac{\beta_3'(M_{\rm GUT})}{{g_3'}^2(M_{\rm GUT})}\right] F_\phi \;, \tag{36}$$

$$m_{\lambda 1}(\mu) \;=\; g_1^2(\mu)\left[\frac{\beta_5(M_{\rm GUT})}{g_5^3(M_{\rm GUT})} + \frac{\beta_1'(M_{\rm GUT})}{15{g_1'}^2(M_{\rm GUT})}\right] F_\phi \;. \tag{37}$$

In the specific model presented in Section 4, $\beta_3' = 0$ (at one loop) and so the gluino mass is also given by the simple GUT prediction.

We now consider scalars. We first assume that D-term contributions to the scalar masses are negligible. In the '5–3–1' model, this is the case for $c - \bar{c} \ll 1$, and $\lambda_2 - \lambda_3 \ll 1$ as discussed in Section 4. Scalar masses then unify into GUT representations at the GUT scale. This is of course familiar in traditional SUGRA models, but does not happen in minimal anomaly mediation, or in the modifications discussed in Refs. [4, 5, 6].

For the first two generations, and for third-generation $\bar{\mathbf{5}}$ fields for small $\tan\beta$, one can neglect Yukawa couplings.[10] The AMSB soft mass at the GUT scale is then given by the gauge contribution

$$m_r^2(M_{\rm GUT}) = -\frac{C_{{\rm GUT},r}}{8\pi^2} g_{\rm GUT}\beta_{\rm GUT}|F_\phi|^2 \;, \tag{38}$$

where $C_{{\rm GUT},r}$ is the Casimir of the GUT representation of the field. (Note that $m_r^2(M_{\rm GUT}) > 0$ if the GUT group is asymptotically free.) The RG equations can then be used to compute the value at the weak scale. The result is

$$m_r^2(\mu) = m_r^2(M_{\rm GUT}) + \sum_{i=1}^{3} \frac{2C_{i,r}}{b_i}\left(\frac{m_{\lambda i}}{g_i^2}\right)^2 \left[g_{\rm GUT}^4 - g_i^4(\mu)\right] \;, \tag{39}$$

where

$$\frac{dg_i}{d\ln\mu} = \frac{b_i g_i^3}{16\pi^2} \;, \tag{40}$$

and $m_{\lambda i}/g_i^2$ is an RG invariant at one loop (given by Eq. (33) for a true GUT, or Eqs. (36) and (37) in the '5–3–1' model). Note that the running down to the weak scale gives a positive contribution to the slepton masses, but the right-handed sleptons get a contribution only from $U(1)_Y$, and this by itself is not large enough to allow

[10] It is possible that these fields have large Yukawa couplings with superheavy fields above the GUT scale. This possibility is unmotivated, and is difficult to reconcile with the requirement that the GUT group is asymptotically free.

the slepton masses at the GUT scale to be significantly negative. This motivates our choice of an asymtotically free GUT group.

For the third generation, the top Yukawa coupling affects the predictions. (For large $\tan\beta$ we must consider the bottom and tau Yukawa couplings as well.) The third-generation scalar masses at the GUT scale are given by

$$m_3^2 \sim +y_t^4 - y_t^2 g_{\rm GUT}^2 + y_t^2\lambda^2 \ , \tag{41}$$

where λ is a cubic coupling involving the GUT Higgs fields. Such couplings are expected generically to be present in order to explain doublet-triplet splitting. We see that there is a model-dependent positive contribution to the third-generation scalar masses, so the only model-independent prediction is a lower bound. The value of y_t is also senstive to $\tan\beta$. Additionally, the μ term is important for the mass eigenstates of the stops (and the sbottoms and staus for large $\tan\beta$) because AMSB gives large A terms. We have checked that for small $\tan\beta$, neglecting the $y_t^2\lambda^2$ contribution at the GUT scale gives third-generation squark masses close to that of the first two generations. We conclude that there is no significant model-independent constraint on the third generation scalar masses, other than the fact that scalar masses of members of the same GUT multiplet unify at the GUT scale.

We now turn to the generic case, in which D-terms cannot be neglected. Different scalars then recieve model dependent contributions to their masses-squared at the GUT scale. For the '5–3–1' model, these contributions are proportional to the hypercharge of the relevant scalar. Eqn. (38) then contains an additional term, given by eqn. (27). In this case, we cannot obtain an analytic expression for the soft masses at low energies. However, as explained in Section 4, in a large region of parameter space, the D-term contribution does not modify the masses significantly, so that the scalar masses-squared at low energies are still positive.

7 Conclusions

We studied grand unified theories (GUT's) in which supersymmetry breaking in the visible sector is communicated by anomaly mediation. We showed that if the GUT scale is determined by the vacuum expectation value of a modulus field, then there is a simple and natural mechanism that fixes the GUT scale at the value

$$M_{\rm GUT} \sim \frac{M_{\rm Planck}}{16\pi^2}, \tag{42}$$

independently of the couplings of the theory. Because the GUT threshold is non-supersymmetric in such models, superpartner masses are not on the anomaly-mediated

trajectory below the GUT scale. This can be used to correct the main phenomenological problem of 'minimal' anomaly-mediated supersymmetry breaking, that is, the tachyonic sleptons. We showed that if the GUT group is asymptotically free one can obtain a viable superpartner mass spectrum at low energies. Also, the resulting superpartner spectrum is very different from that of previously studied AMSB models. In particular, gaugino and scalar masses can unify, subject to corrections from other group factors (for gaugino masses) and D terms (for scalar masses).

We demonstrated these ideas by constructing a GUT model which includes a solution to the doublet-triplet splitting problem. The model we consider has some unappealing features, notably a Landau pole not far from the GUT scale for a $U(1)$ factor. However, we believe it is very interesting that the physics of unification and supersymmetry breaking is so closely intertwined in this class of models. On the one hand, the GUT scale is determined dynamically from the Planck scale by supersymmetry-breaking effects. On the other hand, the GUT physics modifies superpartner masses and leads to a viable low energy theory. We believe that these ideas are worthy of further investigation.

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